

SENSOR

SENSOR TECHNOLOGY LIMITED



- ACOUSTIC TRANSDUCERS
- HYDROPHONES



Piezoelectric Material Constants for Compliance Matrix

January 1990

Revision December 2006

For further technical support, Please contact

Techsupport@sensortech.ca

Tel: +1.705.444.1440

Fax: +1.705.444.6787

Sound Solutions in UNDERWATER SENSORS

WWW.SENSORTECH.CA

ISO 9001:2000
REGISTERED

Established
in 1983

Piezoelectric and electrostrictive materials are important constituents of electromechanical sensors, actuators and smart structures. Piezoelectric materials produce a strain, S , under the influence of an external electric field, E , or become electrically polarised under the influence of an external stress, T . The property of piezoelectricity is closely related to the phenomenon of ferroelectricity, which describes the spontaneous polarisation in a crystal that can be changed between two or more distinct directions with respect to the crystal axes through the application of an external electric field. This ability of ferroelectric materials to switch

polarisation under an external electric field from a random orientation to a preferred direction is used in a variety of polycrystalline ferroelectric materials (ceramics and polymers) to produce a polycrystalline piezoelectric material with a net preferred polarisation direction. This process is described by the term “poling”. Prior to poling, individual domains of the ceramic are piezoelectric but the random orientations counteract each other and the net effect is that the macroscopic material shows little or no piezoelectricity. The partial alignment of the domains during poling creates a net spontaneous polarisation in the poling direction and the material shows a C_{∞} symmetry around that direction.

Piezoelectricity can be mathematically described by a phenomenological model derived from thermodynamic potentials. The derivations are not unique and the set of equations describing the direct and converse piezoelectric effect depend on the choice of potential and the independent variables used. For example, one such set of linear constitutive relations is:

$$S_p = s_{pq}^E T_q + d_{pm} E_m \quad (1)$$

$$D_m = \varepsilon_{mn}^T E_n + d_{pm} T_p$$

where D is the electric displacement, s is the elastic compliance, d is a piezoelectric constant and ε is the dielectric permittivity. The superscripts of the constants designate the independent variable that is held constant when defining the material coefficient and the subscripts define tensor directions which take into account the anisotropic nature of the material. The elements of the tensor form a 9x9 matrix with 1,2,3 designating the orthonormal directions (3 is the poling direction) and 4,5,6 designating the shear directions. For the commonly used polycrystalline piezoelectric ceramic materials with C_{∞} symmetry, such as lead zirconate titanate or PZT, there are ten non-zero, independent matrix elements consisting of 5 independent elastic constants, 3 independent piezoelectric constants and 2 independent dielectric constants. For these materials, the reduced matrix form of the above constitutive relationships can now be written as:

$$\begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \\ D_1 \\ D_2 \\ D_3 \end{bmatrix} = \begin{bmatrix} s_{11}^E & s_{12}^E & s_{13}^E & 0 & 0 & 0 & 0 & 0 & 0 & d_{13} \\ s_{12}^E & s_{11}^E & s_{13}^E & 0 & 0 & 0 & 0 & 0 & 0 & d_{13} \\ s_{13}^E & s_{13}^E & s_{33}^E & 0 & 0 & 0 & 0 & 0 & 0 & d_{33} \\ 0 & 0 & 0 & s_{55}^E & 0 & 0 & 0 & 0 & d_{15} & 0 \\ 0 & 0 & 0 & 0 & s_{55}^E & 0 & d_{15} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2(s_{11}^E - s_{12}^E) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & d_{15} & 0 & \varepsilon_{11}^T & 0 & 0 & 0 \\ 0 & 0 & 0 & d_{15} & 0 & 0 & 0 & \varepsilon_{11}^T & 0 & 0 \\ d_{13} & d_{13} & d_{33} & 0 & 0 & 0 & 0 & 0 & 0 & \varepsilon_{33}^T \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ E_1 \\ E_2 \\ E_3 \end{bmatrix} \quad (2)$$

While the linear constitutive relations can be written in ways other than shown in (1), there are only 10 independent constants and the IEEE Standard on Piezoelectricity contains the appropriate equations that allow one to convert from one set of equations/matrix to another. Ideally, under small fields and stresses and for materials with low losses within a limited frequency range, these 10 constants contain all the information required to predict the behaviour of the material when a stress, strain or electric field is applied to it. However, piezoelectric materials generally exhibit varying degrees of non-linearity. If the above linear equations are used to define the material constants of piezoelectric materials, then the material constants themselves are a function of applied fields and stresses as well as of the temperature of the material. Also, the piezoelectric response is not instantaneous and the response time can be important in some applications.

Piezoelectric and dielectric constants for various piezoelectric materials

		BM500	BM400	BM800
S_{11}^E	10-12 m²/N	15.5	12.5	11.0
S_{12}^E		-5.0	-4.0	-3.5
S_{13}^E		36.0	6.0	-5.0
S_{33}^E		19.0	15.0	13.5
S_{55}^E		45.0	40.0	30.0
d_{13}	10⁻¹² C/N	-175.0	-125.0	-60.0
d_{33}		365.0	300.0	225.0
d_{15}		585.0	480.0	285.0
ϵ_{11}^T		1,650.0	1,450.0	1,100.0
ϵ_{33}^T		1,750.0	1,350.0	1,000.0